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LETTER TO THE EDITOR

Transport and elementary excitations of a Luttinger liquid

Gianaurelio Cuniberti†, Maura Sassetti‡ and Bernhard Kramer§

† Dipartimento di Fisica, INFM, Università di Genova, Via Dodecaneso 33, I-16146 Genova, Italy

‡ Istituto di Fisica Ingegneria, INFM, Università di Genova, Via Dodecaneso 33, I-16146 Genova, Italy

§ I. Institut für Theoretische Physik, Universität Hamburg, Jungiusstraße 9, D-20355 Hamburg, Germany

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Abstract. The low-temperature AC conductance of a one-dimensional electron system with a strong interaction of finite range is calculated by using linear response theory. The conductance factorizes into parts which depend on the internal properties of the system, and the external probe. For short-range interaction, the result resembles that for non-interacting electrons, but with the zero-frequency limit and the Fermi velocity renormalized by the interaction strength. For strong and long-range interaction, the conductance shows a peak that is related to charge-wave excitations. In this limit, the AC conductance can be simulated by a *quantum capacitance* and a *quantum inductance*.

Recently, frequency- and time-dependent electrical transport processes in nanostructures have become a subject at which increasing experimental effort has been directed [1, 2]. Since they provide insight into the elementary excitations of these systems of geometrically confined interacting electrons, such investigations are of great fundamental interest. In addition, potential applications of nanostructures in future electronic devices, which will have to be operated at very high frequencies, require detailed knowledge of their AC-transport behaviour.

The theory of AC quantum transport has been mostly restricted to non-interacting electrons [3, 4], and to driven systems [5, 6]. Coulomb repulsion was partly taken into account by including a classical charging term [7]. On the other hand, in the DC transport through quantum dots, correlation effects were shown to be of great importance [8]. In view of the quantum nature of the systems, which has to be properly taken into account in transport theory, the study of the AC-transport properties of a *Luttinger model* should be of considerable interest [9], since the latter is a paradigmatic example of a correlated electron system. We explore in this paper the linear AC transport in the Luttinger system with a *long-range interaction*.

We show that the conductance, $\Gamma(\omega)$, is a product of two functions. One of them is the inverse of the derivative of the dispersion relation of the collective excitations. The other is given by the Fourier transform of the applied electric probe field. Our result implies that a short-range interaction does not lead to qualitative changes in the the behaviour of $\Gamma(\omega)$ as compared with the non-interacting limit. Only its magnitude, and the Fermi velocity, v_F, are scaled by g and g^{-1}, respectively, where g^{-2} is the interaction strength. For strong and long-range interaction, the presence of a plasmon-like charge-wave mode in the dispersion relation [10] leads to a resonance in $\Gamma(\omega)$ at $\omega_p \propto v_F/\lambda g$, where λ is the 'effective range' of the interaction.

We will see below that the resonance is independent of the shape of the electrical probe field if the range of the latter $\ell < \lambda$. The resonance should be observable in an experiment. In fact, Raman scattering measurements on quantum wires [11] were interpreted within such a picture. Here, we want to point out that one can detect the resonance at temperatures lower than ω_p by an *AC-conductance experiment*. This would provide evidence for the Luttinger-liquid nature of electrons in quantum wires [12] or of the edge states in the fractional quantum Hall [6] effect without relying on determinations of temperature dependences.

In addition, we show that for *sufficiently strong and long-range interaction* the results can be understood in terms of a *quantum capacitance* and a *quantum inductance*, $C \equiv C_0\, g^2\, \lambda$, $L \equiv L_0\, \lambda$, in analogy with a classical wire (C_0, L_0 constants). For the microscopic understanding of the Coulomb blockade effect [13] in submicrometre tunnel contacts such quantities are of crucial importance. In previous theories, they were introduced phenomenologically. The important message of this letter is that for a system to show capacitive behaviour, the interaction should be *long range*.

The Luttinger liquid is a model for the *low-energy excitations* of one-dimensional (1D) interacting electrons [14]. Its major importance is that the excitation spectrum can be calculated analytically, as can many other properties, like the linear conductivity, even in the presence of perturbing potentials [9]. The main assumptions are: (1) linearization of the free-electron dispersion relation near the Fermi level; and (2) extension of the energy spectrum to include negative energies. For spinless particles, the subject of this letter, with interaction $V(x) = V_0\,\delta(x)$ and neglecting backward scattering, the Hamiltonian is

$$H = \frac{\hbar v_F}{2g}\int_0^R \mathrm{d}x \left(g\,\Pi^2(x) + \frac{1}{g}\left(\frac{\partial}{\partial x}\Theta(x)\right)^2\right) \tag{1}$$

with R and $g = (1 + V_0/\hbar\pi v_F)^{-1/2}$ the length of the system and the coupling parameter, respectively. The latter decreases with increasing interaction strength. For $g = 1$ the interaction vanishes, and $g < 1$ and $g > 1$ correspond to repulsive and attractive interaction, respectively. The fields Π and Θ are conjugate to each other: $[\Theta(x), \Pi(x')] = \mathrm{i}\delta(x - x')$.

Hamiltonian equation (1) is easily diagonalized by introducing bosonic operators, b_k, $b_k^\dagger$ [14]. One finds

$$H = \hbar \sum_k \omega(k) b_k^\dagger b_k$$

with $\omega(k) = v_F|k|/g$. For a general interaction $V(x - x')$ (Fourier transform $V(k)$) the Hamiltonian can be diagonalized by a Bogoliubov transformation [10, 14]. The resulting dispersion relation is

$$\omega(k) = v_F|k|\sqrt{1 + V(k)/\hbar\pi v_F}\,. \tag{2}$$

Apparently, $v_F\sqrt{1 + V(k)/\hbar\pi v_F}$ plays the role of a (k-dependent) 'charge-wave' velocity.

We evaluated $\omega(k)$ for $V^L(x) = V_0^L \alpha \mathrm{e}^{-\alpha|x|}/2$ and $V^C(x) = V_0^C \mathrm{e}^{-\alpha\sqrt{x^2+d^2}}/\sqrt{x^2 + d^2}$ with $V_0^L = e^2/\varepsilon_0(\alpha d)^2$ and $V_0^C = e^2/4\pi\varepsilon_0$, respectively. These may be obtained from a screened 3D Coulomb interaction (screening length α^{-1}) by calculating for $\alpha d \gg 1$ and $\alpha d \ll 1$ the effective interaction potential of particles, parabolically confined within an infinitely long quantum wire of diameter d. When $\alpha \to \infty$, $V^L(x) \to V_0^L\,\delta(x)$, thus recovering the Luttinger limit, equation (1). When $\alpha \to 0$, $V^C(x) = V_0^C/\sqrt{x^2 + d^2}$ is the unscreened effective Coulomb interaction. The corresponding Fourier transforms are $V^L(k) = V_0^L\,\alpha^2/(k^2 + \alpha^2)$ and $V^C(k) = V_0^C\, 2K_0(d\sqrt{k^2 + \alpha^2})$ (K_0 is a Bessel function).

The dispersion relations for V^C are shown in figure 1 for different coupling strengths and $\alpha d = 10^{-3}$.

When $kd \ll 1$, $\omega(k) = |k|v_F/g_C$, where the strength of the interaction is $g_C^{-2} \equiv 1 + 2V_0^C K_0(\alpha d)/(\hbar\pi v_F)$. The velocity of this 'charge sound wave' is strongly enhanced by the interaction as compared to v_F. For $kd \gg 1$, $\omega(k) = |k|v_F$—that of non-interacting electrons—since $K_0 \approx 0$ (for the spinless electrons that we consider here). In the intermediate region, $kd = \mathrm{O}(1)$, there is a crossover between the two asymptotic dispersions such that $\omega'(k) \equiv \mathrm{d}\omega(k)/\mathrm{d}k$ has a minimum at some finite wave number k_p, i.e. $\omega''(k_p) = 0$. We interpret $\omega_p = \omega(k_p)$ as a 'plasmon frequency'. However, in contrast to the conventional plasmon frequency, it corresponds to excitations with a *non-zero wave vector* k_p.

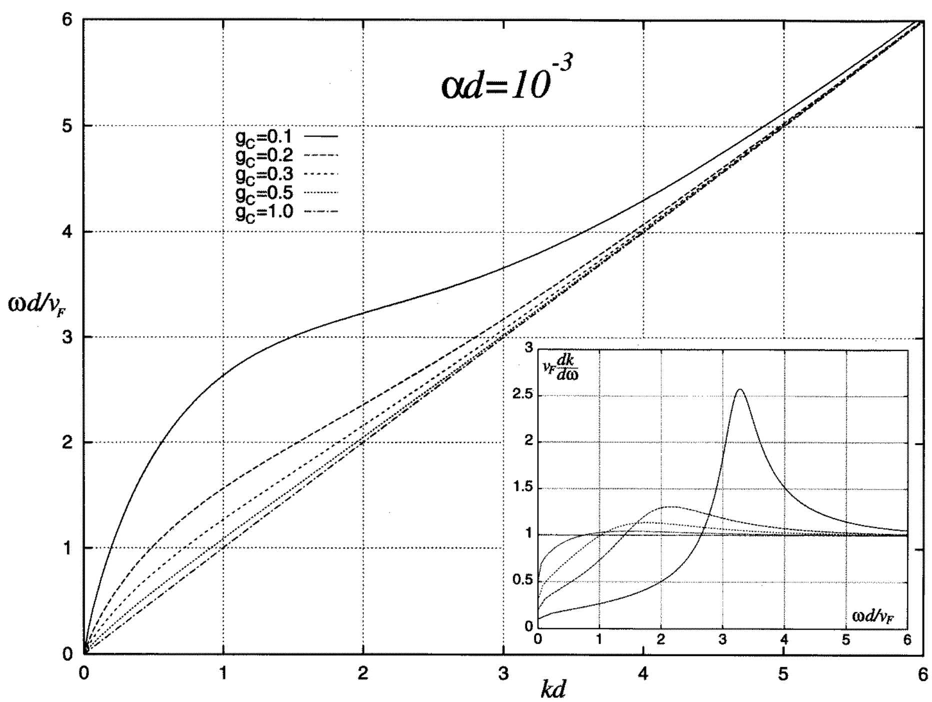

Figure 1. The dispersion relation $\omega(k)$ of the Luttinger model with screened Coulomb interaction for different g_C and $\alpha d = 10^{-3}$. Inset: the derivative $\mathrm{d}k/\mathrm{d}\omega$

By a straightforward calculation, starting from the observation (cf. figure 1) that $k_p d = \mathrm{O}(1)$ one obtains for $g_C \ll 1$ (strong interaction)

$$\omega_p^2 = \frac{v_F^2 a^2}{g_C^2 K_0(\alpha d) d^2} \approx \frac{v_F^2 a^2}{d^2} \frac{2V_0^C}{\hbar\pi v_F} \equiv \frac{v_F^2 a^2}{g_0^2 d^2} \tag{3}$$

where

$$a \equiv k_p d \sqrt{K_0(k_p d)\left[1 + g_C^2\left(K_0(\alpha d)/K_0(k_p d) - 1\right)\right]}$$

depends only weakly on g_C. The dispersion relation for V^L is qualitatively very similar. One obtains $\omega_p \approx v_F\,\alpha/g_L$, with $g_L^{-2} \equiv 1 + V_0^L/(\hbar\pi v_F) \ll 1$. The corresponding wave vector is $k_p \approx 3^{1/4}\alpha/\sqrt{g_L}$, for $g_L \ll 1$. The flat part of $\omega(k)$ at intermediate wave numbers implies the presence of a peak in the derivative of the inverse of the dispersion, $k(\omega)$ (figure 1). We show now that this can be directly observed in the AC conductance.

In equation (1), the field $\partial\Theta(x)/\partial x$ represents the density. The current density, $j(x,t)$, may be obtained via the continuity equation

$$j(x,t) = -\frac{e}{\sqrt{\pi\hbar}}\frac{\partial}{\partial t}\Theta(x,t). \tag{4}$$

The *absorptive part* of the non-local quantum mechanical conductivity, $\sigma(x,x';\omega)$, is given by the Kubo formula. After some straightforward calculations one gets

$$\sigma(x,x';\omega) = \frac{v_F e^2}{h}\int_0^\infty \mathrm{d}k\ \cos k(x-x')\left(\delta(\omega(k)+\omega)-\delta(\omega(k)-\omega)\right). \tag{5}$$

The conductance, $\Gamma(\omega)$, is obtained from the conductivity by calculating the absorbed power when an electric probe field, $E(x)$, is applied [3]:

$$\Gamma(\omega) = \frac{v_F e^2}{h}\int_0^\infty \mathrm{d}k\ L(k)\left(\delta(\omega(k)+\omega)-\delta(\omega(k)-\omega)\right). \tag{6}$$

Here,

$$L(k) \equiv \left|\int_{-\infty}^{\infty}\mathrm{d}x\ \mathrm{e}^{\mathrm{i}kx}E(x)\right|^2 \Big/ U^2$$

with the voltage

$$U \equiv -\int_{-\infty}^{\infty}\mathrm{d}x\ E(x).$$

For a monotonic dispersion (cf. figure 1), we find

$$\Gamma(\omega) = \frac{e^2 v_F}{h}\left(\frac{\mathrm{d}\omega}{\mathrm{d}k}\right)^{-1}_{\omega(k)} L(k(\omega)). \tag{7}$$

For a δ-interaction $\Gamma(\omega) = (ge^2/h)L(g\omega/v_F)$—the same as without interaction [3], except for the renormalization of the prefactor and the Fermi velocity with g. For an interaction potential of finite range we get asymptotically $\Gamma(\omega) = (g_{C,L}e^2/h)L\left(g_{C,L}\omega/v_F\right)$, and $\Gamma(\omega) = (e^2/h)L(\omega/v_F)$, for $\omega \to 0$ and $\omega \to \infty$, respectively, due to the limiting behaviour of $\omega(k)$ for small and large $|k|$.

The most important feature of (7) is that there is a separation between the internal properties of the system, represented by $\mathrm{d}k(\omega)/\mathrm{d}\omega$, and the external probe field, represented by $L(k)$—the Fourier transform of its spatial autocorrelation function [3]. The result (7) implies that internal properties of a mesoscopic quantum system can be determined, provided that the spatial properties of the probe field are known. If the latter is constant within an interval ℓ, $L(k) = \sin^2 k\ell/(k\ell)^2$. For small $k\ell$, this is essentially a constant. There are zeros for $k\ell = n\pi$. If $E(x)$ is a smooth function around $x = 0$, rapidly decaying for $|x| \to \infty$, $L(\omega)$ will also be a rapidly decaying function, without any particular structure. Then, the features of the dispersion of the elementary excitations are directly displayed by the AC conductance provided that $\omega_p < \omega_1$. The frequency $\omega_1 = \omega(\pi/\ell)$ corresponds to the width in frequency of the Fourier transform of the probe field. It is given by equation (2) when $k = \pi/\ell$. In order to observe the resonance experimentally one should have $\ell/d < 1$. Near-field optical spectroscopy [15] should in principle be able to provide the basis for a technique for making such a measurement.

We conclude by mentioning that the above results suggest defining a *capacitance* and an *inductance* of a quantum wire when the interaction is strong *and* long range. We concentrate on the Coulomb limit. The Luttinger limit can be treated analogously [16].

Consider the classical capacitance, C, and inductance, L, of a charge and a current distribution via electrostatic and magnetic energy [17], respectively. For a 'wire' (of length R ($\to\infty$), with charge and current density $\propto \exp\left(-(y^2+z^2)/2d^2\right)$), they are $Q^2/2C \equiv (4\pi\varepsilon_0)^{-1}Q^2R^{-1}\ln R/d$, and $LI^2/2 \equiv (\mu_0/4\pi)I^2R\ln R/d$, respectively. Here, Q and I are the total charge and current, respectively. Thus $C = 2\pi\varepsilon_0 R/\ln(R/d)$ and $L = (\mu_0/2\pi)R\ln(R/d)$. The corresponding resonance frequency is $\omega_0^2 \equiv (LC)^{-1} = (\varepsilon_0\mu_0)^{-1}R^{-2}$.

Like the plasma frequency ω_p with wave vector $k_p \propto \pi/d$, ω_0 represents a resonance of a mode with wave number $k_R = \pi/R$ of the classical wire. This suggests replacing R by d in ω_0 when attempting to translate the classical resonance frequency into ω_p. The result, equation (3), can then be reproduced by making the additional identifications

$$\varepsilon_0 \to \frac{e^2}{h}\frac{g_C^2 K_0(\alpha d)^2}{v_F a^2} \approx \frac{e^2}{h}\frac{g_0^2 K_0(\alpha d)}{v_F a^2} \qquad \mu_0 \to \frac{h}{e^2}\frac{1}{K_0(\alpha d)v_F}\,. \tag{8}$$

This choice is, of course, not unique. It is motivated by the fact that $e/\sqrt{\varepsilon_0}$ represents the interaction strength in the electrostatic energy. The product of ε_0 and μ_0 should be independent of αd. Thus, the extra factor $K_0(\alpha d)$ when replacing ε_0 requires a factor K_0^{-1} when replacing μ_0. The latter are necessary in order to compensate the logarithmic terms in C and L which result from the cut-off at R in the integrations for the electrostatic and magnetic energies. In the microscopic theory, the cut-off is provided by the screening length α^{-1}. Therefore, we choose to replace $\ln(R/d)$ in the classical expressions for C and L by $K_0(\alpha d)$ ($\propto -\ln\alpha d$) for $\alpha d \to 0$. The prefactor e^2/hv_F appears for dimensional reasons.

The *quantum capacitance* and *quantum inductance* of the Luttinger 'wire' can then be defined as

$$C_q \equiv \frac{e^2}{h}\frac{2\pi}{v_F a^2}g_0^2 d \qquad L_q \equiv \frac{h}{e^2}\frac{1}{2\pi}\frac{d}{v_F}\,. \tag{9}$$

As the *quantum conductance*, $\Gamma_q \equiv e^2 g_C/h$, C_q and L_q are independent of the wire length. They depend only on microscopic properties. While Γ_q vanishes as $|\ln\alpha d|^{-1/2}$ for $\alpha \to 0$ (Coulomb interaction), the latter stay finite. This implies that the AC-transport behaviour of the Luttinger wire can also be simulated by a classical circuit of an inductance, a capacitance and resistances, and leads to a generalization of the above results which accounts for the finite width of the resonance in the AC conductance [16].

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